



Figure 19.0.3. Matrix structure derived from a second-order elliptic equation (here equation 19.0.6). All elements not shown are zero. The matrix has diagonal blocks that are themselves tridiagonal, and sub- and super-diagonal blocks that are diagonal. This form is called “tridiagonal with fringes.” A matrix this sparse would never be stored in its full form as shown here.

Relaxation methods make immediate use of the structure of the sparse matrix

A. The matrix is split into two parts

$$\mathbf{A} = \mathbf{E} - \mathbf{F} \tag{19.0.12}$$

where \mathbf{E} is easily invertible and \mathbf{F} is the remainder. Then (19.0.10) becomes

$$\mathbf{E} \cdot \mathbf{u} = \mathbf{F} \cdot \mathbf{u} + \mathbf{b} \tag{19.0.13}$$

The relaxation method involves choosing an initial guess $\mathbf{u}^{(0)}$ and then solving successively for iterates $\mathbf{u}^{(r)}$ from

$$\mathbf{E} \cdot \mathbf{u}^{(r)} = \mathbf{F} \cdot \mathbf{u}^{(r-1)} + \mathbf{b} \tag{19.0.14}$$

Since \mathbf{E} is chosen to be easily invertible, each iteration is fast. We will discuss relaxation methods in some detail in §19.5 and §19.6.

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